Q1. LU Decomposition.

Perform the LU decomposition of the following 2×2 matrix A:

$$A = \begin{bmatrix} 4 & 3 \\ 8 & 11 \end{bmatrix}$$

Show the step-by-step analytical derivation for both matrices L and U such that $A = LU$.

Q2. Taylor Expansion.

For the exponential function $f(x) = e^x$:

(a) Derive the polynomial approximation terms explicitly up to the fourth-order term about $x = 0$.

(b) Express the infinite series in general summation notation.

Q3. Simple Harmonic Oscillator.

Solve the differential equation for a simple harmonic oscillator:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

subject to the initial condition $x(0) = x_0$ and $\frac{dx}{dt}(0) = v_0$ and $\omega > 0$ is the angular frequency.

Q4. Moving Average Filter.

Consider a 3-point causal moving average filter defined by the system difference equation:

$$y[n] = \frac{1}{3}\big(x[n] + x[n-1] + x[n-2]\big)$$

where $x[n]$ is the input sequence and $y[n]$ is the smoothed output sequence.

(a) Given an input time-series signal $x[n]$ defined for $n \geq 0$: $x = \{3, 9, 6, 12, 9\}$, calculate the output values $y[0]$, $y[1]$, and $y[2]$ explicitly. Assume causal initial conditions where $x[n] = 0$ for all $n < 0$.

(b) Answer Yes/No:

		Yes	No
(b-1)	Does this filter attenuate sudden, high-frequency noise spikes in a signal?	☐	☐
(b-2)	Does this filter allow slowly varying, low-frequency trends to pass through largely unaffected?	☐	☐
(b-3)	Is this moving average filter classified as a low-pass filter?	☐	☐