

Q1. Interpolation.

We are given samples $(x_1, y_1), \dots, (x_k, y_k)$ and look for an interpolant $f(x) = \sum_{i=1}^k c_i \phi_i(x)$ with $f(x_i) = y_i$.

(a) Answer Yes/No:

		Yes	No
(a-1)	Do the monomial, Lagrange, and Newton bases give three <i>different</i> polynomials through the same k points?	☐	☐
(a-2)	In piecewise linear interpolation, does changing a single value y_i change the interpolant only on the intervals next to x_i ?	☐	☐

(b) Consider the two samples $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (2, 7)$.

- Write out the two Lagrange basis functions ϕ_1, ϕ_2 and the interpolant $f(x) = \sum_i y_i \phi_i(x)$.
- Set up the lower triangular system for the Newton basis $1, (x - x_1)$, and solve it for the coefficients.
- Expand both results and show that they are the same polynomial.

Q2. Numerical Integration.

We approximate $\int_a^b f(x) dx$ by a quadrature rule $Q[f] = \sum_{i=1}^n w_i f(x_i)$.

(a) Answer Yes/No:

		Yes	No
(a-1)	Is Gaussian quadrature a suitable choice for integrating a recorded audio signal whose samples are on a fixed uniform grid?	☐	☐
(a-2)	Does the error of Monte Carlo integration decay like $N^{-1/2}$ regardless of the dimension?	☐	☐

(b) Take the nodes $x_1 = 0$ and $x_2 = 2$ of **Q1(b)** as the end points of the interval $[a, b] = [0, 2]$.

- Replace f by its interpolant $\sum_i f(x_i) \phi_i(x)$ and integrate it over $[0, 2]$ to show that the weights of the resulting rule are $w_i = \int_0^2 \phi_i(x) dx$.
- Compute w_1, w_2 with the Lagrange basis functions of **Q1(b)**. Which rule from the lecture is this?
- Apply the rule to the samples of **Q1(b)**, and compare with the exact integral of the polynomial found there.