

Q1. Numerical Differentiation.

We approximate $f'(x)$ from two values of f at distance h , using the Taylor series

$$f(x \pm h) = f(x) \pm hf'(x) + \frac{h^2}{2}f''(x) \pm \frac{h^3}{6}f'''(x) + \cdots$$

Consider the forward and centered differences

$$\delta_+ = \frac{f(x+h) - f(x)}{h}, \quad \delta_c = \frac{f(x+h) - f(x-h)}{2h}.$$

- Use the Taylor series to show that $\delta_+ = f'(x) + \frac{h}{2}f''(x) + \cdots$ and $\delta_c = f'(x) + \frac{h^2}{6}f'''(x) + \cdots$. What is the order of accuracy of each?
- Take $f(x) = x^3$, $x = 1$, and $h = 0.1$. Compute δ_+ and δ_c , and compare their errors with the leading terms above.
- If h is halved, by roughly what factor does each error shrink?

Q2. Automatic Differentiation.

(a) Answer Yes/No:

		Yes	No
(a-1)	Does automatic differentiation have a step size h to tune, like finite differences?	☐	☐
(a-2)	For a loss L of a network with 10^6 weights θ , does one backward pass over the tape give all 10^6 partial derivatives $\partial L / \partial \theta_i$?	☐	☐
(a-3)	Can the backward pass discard the intermediate values computed during the forward run?	☐	☐
(a-4)	Does training a physics-informed network for the wave equation always require a reference solution or recorded data?	☐	☐